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Optimizing an Advertising Campaign

Math 1010 Project

Background Information:

Linear Programming is a technique used for optimization of a real-world situation. Examples of optimization include maximizing the number of items that can be manufactured or minimizing the cost of production. The equation that represents the quantity to be optimized is called the objective function, since the objective of the process is to optimize the value. In this project the objective is to maximize the number of people who will be reached by an advertising campaign.

The objective is subject to limitations or constraints that are represented by inequalities. Limitations on the number of items that can be produced, the number of hours that workers are available, and the amount of land a farmer has for crops are examples of constraints that can be represented using inequalities. Broadcasting an infinite number of advertisements is not a realistic goal. In this project one of the constraints will be based on an advertising budget.

Graphing the system of inequalities based on the constraints provides a visual representation of the possible solutions to the problem. If the graph is a closed region, it can be shown that the values that optimize the objective function will occur at one of the "corners" of the region.

The Problem:

How many of each type of ad (radio and TV) should be purchased to maximize the number of people who will be reached by the advertisements?

The Information:

A local business plans on advertising their new product by purchasing advertisements on the radio and on TV. Here is the information the business needs to use to make the best decision.

a. The business plans to purchase at least 80 total ads.

b. Radio ads cost $200 each.

c. TV ads cost $800 each.

d. The total advertising budget for these ads is $59,800.

e. They want to have at least three times as many TV ads as radio ads (i.e. there are MORE TV ads than radio ads).

f. It is estimated that each radio ad will be heard by 1,900 listeners

g. It is estimated that each TV ad will be seen by 14,500 people.

Modeling the Problem:

Let X be the number of radio ads that are purchased, and Y be the number of TV ads.

1. Write down a linear inequality for the total number of desired ads.

$x + y \geq 80$

2. Write down a linear inequality for the cost of the ads.

$200x + 800y \leq 59{,}800$

3. Recall that the business wants at least three times as many TV ads as radio ads. Write down a linear inequality that expresses this fact. (e.g. if there are 10 radio ads there should be ***at least*** 30 TV ads).

$y \geq 3x$

4. There are two more constraints that must be met. These relate to the fact that there cannot be negative numbers of advertisements. Fill in the two inequalities that model these constraints:

$X \geq$ 0

$Y \geq$ 0

5. Next, write down the function for the number of people that will be exposed to the advertisements. This is the Objective Function for the problem and is a function of both X and Y.

$P = 1900x + 1450y$

You now have five linear inequalities and an objective function. These together describe the situation. This combined set of inequalities and objective function make up what is known mathematically as a **linear programming** problem.

6. To determine the number of ads, you will need to graph the **intersection** of all five inequalities on one common XY plane (Do not plot the objective function!). Use Desmos.com to plot the inequalities. Scale your axes (or zoom in) so that you have the bottom left be the origin, with the horizontal axis representing X and the vertical axis representing Y. Include your graph with your assignment by either taking a screen shot of it or using the print feature on Desmos. Label the points of intersection on your graph as well.

7. The shaded region in the above graph is called the **feasible region**. Any (x, y) point in the region corresponds to a possible number of radio and TV ads that will meet all the requirements of the problem. However, the values that will **maximize** the number of people exposed to the ads will occur at one of the vertices or corners of the region. Your region should have three corners.

   Find the coordinates of these corners by solving the appropriate systems of linear equations. Be sure to *show your work* and *label* the (x, y) coordinates of the corners in your graph.

1) $x+y \geq 80$

$y \geq 3x$

$x+3x=80$

$4x=80$

$\frac{4x}{4} = \frac{80}{4}$

$x=20$

$20+y=80$

$y=60$

$(20, 60)$

2) $x+y \geq 80$

$200x+800y \leq 59{,}800$

$x=80-y$

$200(80-y)+800y=59{,}800$

$1600+600y=59000$

$600y=43800$

$\frac{600y}{600} = \frac{43800}{600}$

$y=73$

$x+73=80$

$x=7$

$(7, 73)$

3) $200x+800y \leq 59{,}800$,

$y \geq 3x$

$200x+800(3x)=59800$

$200x+2400x=59{,}800$

$2600x=59800$

$\frac{2600x}{2600} = \frac{59800}{2600}$

$x=23$

$y=3(23)$

$y=69$

$(23, 69)$

8. To find which combination of radio and TV ads will maximize the number of people who are exposed to the business advertisements, evaluate the objective function P for each of the vertices you found. Show your work.

i. Vertex: P=1900(20)+14500(60) = 908,000
38,000 + 870000

ii. Vertex: P = 1900(7) + 14500(73) = 1,071,800
13300 +

iii. Vertex: P = 1900(23) + 14500(69)
43700 + 1000500 = 1,044200

9. Write a sentence or two describing your conclusion. Include in your statement: how many of each type of advertisement should be purchased and what is the maximum number of people who will be exposed to the ads.

to maximize the number of people reached the business should purchase 7 radio ads and 73 tv ads to reach a maximum of 1,071,800 people.

10. **Reflective Writing**.

This needs to be a separate page that is typed, proof-read for typos, spelling, and grammar. Your writing should be in an essay form (written in paragraphs). Use 12-point font and double space. Add a title (e.g. Reflective Writing for Optimization Project). Your instructor will provide details for turning it in.

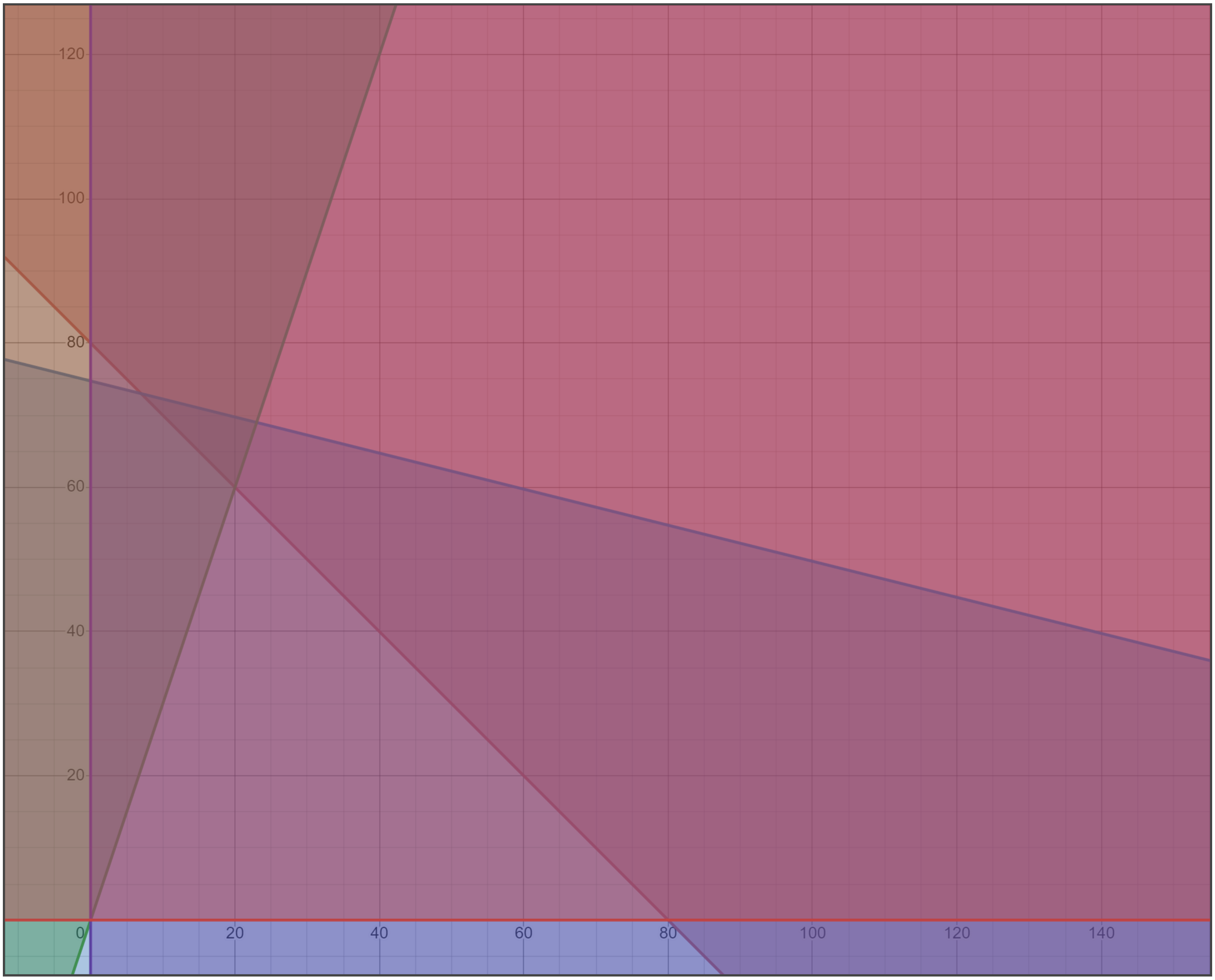
Provide a brief introduction explaining the lab in your own words. Also in the introduction, tell the audience which mathematical techniques you used in the lab (e.g. linear equations, etc.). Then, please respond to each of the questions (in essay form, so paraphrase the question you are answering).

- Can you give an example of another application where this type of analysis would be beneficial? Be specific.
- If you worked for an advertising agency, why would it be important to be able to explain the details of this project to clients?
- Discuss in your own words how the results might change if the given information (i.e. numbers of ads, costs, budget) changed.

- How would you interpret a result if the vertices weren’t at integer values? For example, what if you found 15.2 radio ads and 46.8 TV ads? What would you do?
- Did this assignment change your opinion of the usefulness of math? Write one paragraph stating what ideas changed and why. If this project did not change the way you think, write how this project gave further evidence to support your existing opinion about applying math. Be specific.

ePortfolio:

Post a copy of this lab, including the Reflective Writing, to your ePortfolio (you can print a digital version as a pdf, or if you need to, scan a copy in the Copy Center). For more information about ePortfolios, please see the syllabus.



1 $x + y \geq 80$

2 $200x + 800y \leq 59800$

3 $y \geq 3x$

4 $x \geq 0$

5 $y \geq 0$

6

7

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Reflective Writing for Optimization Project

In this project, I utilized linear programming techniques to optimize an advertising campaign for a local business. By using linear inequalities and an objective function, I aimed to maximize the number of people reached by the advertisements. Through graphical analysis and vertex evaluation, I graphed the feasible region and identified the optimal solution.

The mathematical techniques used in this project included the formulation of linear inequalities to represent constraints and a function to quantify the goal. By graphing these inequalities and evaluating them at various points, I employed graphical analysis to visualize the feasible region and identify the vertices that represent optimal solutions.

The benefit of this analysis extends beyond advertising to various industries, such as supply chain management. Optimization in production, distribution, and inventory management.

Understanding the details of this project is crucial for advertising agencies to provide effective recommendations to clients. By ensuring that advertising campaigns maximize reach within budget constraints, agencies can deliver better results.

Changing information, such as variations in the number of ads, costs, or budget, can alter the best solution and maximum reach. Therefore, it is essential to continuously re-evaluate and adjust strategies based on updated information.

Interpreting non-integer results is an important aspect of real-world applications. While vertices may not always align with whole numbers, fractional quantities of ads can still be meaningful.

This assignment underscored the practical usefulness of mathematics in solving real-world problems. By providing a structured approach to optimization, it demonstrated how mathematical techniques can inform decision-making and achieve desired outcomes. As a result, my appreciation for the practical applications of mathematics has been further solidified.